LITTORAL MOCK

General Certificate of Education Examination

MARCH 2019 **ADVANCED LEVEL**

Subject Title	**Further Mathematics**
Paper number	**2**
Subject Code	**0775**

THREE HOURS

INSTRUCTIONS TO CANDIDATES

Answer ALL questions.

For your guidance the approximate mark allocation for parts of each question is indicated bold.

Mathematical formulae and tables, published by the Board, and noiseless non – programmable electronic calculators are allowed.

In calculations you are advised to show all the steps in your working giving your answer at each stage.

1. Using the definition $\tanh y = \dfrac{e^{y} - e^{-y}}{e^{y} + e^{-y}}$,

a) show that , for $|x| < 1$, $\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$ **4 marks**

b) Hence, or otherwise, show that $\dfrac{d}{dx}\left(\tanh^{-1} x\right) = \dfrac{1}{1-x^{2}}$ **3 marks**

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2. (i) Sketch, using the same axes, the curves $y^{2} = 4x$ and $y^{2} = x^{3}$.

Shade the region for which $\left(y^{2} - 4x\right)\left(y^{2} - x^{3}\right) \leq 0$. **4 marks**

(ii) Solve the differential equation $(x+1)\dfrac{dy}{dx} - 3y = (x+1)^{5}$, given that $y = \dfrac{3}{2}$ when x = 0 **4 marks**

3.(i) The arc of a curve with equation $y = \cosh x$, between the points where $x = 0$ *and* $x = \ln\left(\dfrac{9}{4}\right)$, is rotated through 2π radians about the $x-axis$. Show that the surface area generated is $\pi\left[\ln\left(\dfrac{9}{4}\right) + p\right]$, where p is a number to be determined **5 marks**

(ii) Sketch the graph of the function $f(x) = \left|x^{2} - 4\right| + 3$

Hence, find the solution of the inequality $\left|x^{2} - 4\right| > 2$ **4 marks**

4.(i) The Cartesian equation of the curve C is

$$x^{2} + y^{2} - 2x = \left(x^{2} + y^{2}\right)^{\frac{1}{2}}$$

a) Find the polar equation of the curve for $-\pi < \theta \leq \pi$ **2 marks**

b) Find the equations of the tangents to the curve at the Pole **2 marks**

c) Sketch the curve C. **2 marks**

(ii) Given that $e^{-x} + \dfrac{2x+3}{x+2} \leq f(x) \leq \dfrac{3-7x+2x^{2}}{x+2}$, find $\lim\limits_{x \to +\infty} f(x)$ **3 marks**

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5. (i) When the number 900 is divided by another number b , it gives a quotient 14 and a remainder r .

a) Write down the relationship between 900, 14, b and r, **1 mark**

b) What are the possible values of b and r **5 marks**

(ii) A linear transformation, T, maps the points (1, 1) onto the point (2, - 3) and the point (0, 1) on to (1, 2) .

Find $T\begin{pmatrix} a \\ b \end{pmatrix}$ and hence $T\begin{pmatrix} 3 \\ -19 \end{pmatrix}$ **4 marks**

6.(i) Find the equation of the tangent and normal of the parabola $x^2 = 8y$ at the point $\left(4t, 2t^2\right)$. The tangent meets the x – axis at A and the y – axis at B.

a) Find the coordinates of the mid-point M of AB **4marks**

b) Hence find the locus of M as t varies. **3 marks**

(ii) If $I_n = \int (\ln x)^n \, dx$, show that $I_n = x\ln x - nI_{n-1}, n \geq 1$

Hence find I_3 **4 marks**

7.(i) Prove that $(\mathbb{R}, \circ)$ is a group where $\circ$ is defined by $a \circ b = a + b - 1$ **3 marks**

(ii) Define a map from $\left(\mathbb{R}^*, \times\right)$ *to* $(\mathbb{R}, +)$ by $f(x) = \ln x$, *where* $\mathbb{R}^*$, is the set of positive non-zero real numbers.

a) Show that f is a homomorphism. **3 marks**

b) Show, also that, f is an isomorphism. **3 marks**

8.(i) Determine the $kelf$ *and* $\operatorname{Im} f$ in the linear transformation

$$f : \mathbb{R}^2 \to \mathbb{R}^2$$
$$(x, y) \mapsto (2x+, x - y)$$

5 marks

(ii) Given that $\dfrac{z-1}{z-2} = e^{i\theta}$, *where* θ is real, prove that $z = \dfrac{1}{2}\left[3 - i\cot\left(\dfrac{\theta}{2}\right)\right]$ **4 marks**

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9. A function f, is defined by

$$f(x) = \begin{cases} \dfrac{1}{2} x e^{\frac{1}{x}}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

a) State the domain of f **1 mark**

b) Show that f is not continuous at the point where $x = 0$ **2 marks**

c) Study the differentiability of f and state its sign in the domain **3 marks**

d) Find $\lim\limits_{x \to +\infty} f(x)$ *and* $\lim\limits_{x \to -\infty} f(x)$ **3 marks**

e) State the asymptote of f **1 mark**

f) Establish a variation table for f **2 marks**

g) Interpret your results graphically. **2 marks**

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10. Consider the sequence (u_n) given by $\begin{cases} u_0 = -1, \quad u_1 = \dfrac{1}{2} \\ u_{n+2} = u_{n+1} - \dfrac{1}{4}u_n \end{cases}$

a) By letting and (w_n) be two other sequences defined by $v_n = u_{n+1} - \frac{1}{2}u_n$ and $w_n = \frac{u_n}{v_n}$

i. Calculate v_0 and w_0 **2 marks**

ii. Show that the sequence (v_n) is a geometric progression with common ratio $r = \frac{1}{2}$. **2 marks**

iii. Express v_n in terms of n and hence evaluate $\lim\limits_{n\to\infty} v_n$ **2 marks**

iv. Show that $w_{n+1} = w_n + 2$ and then characterize the sequence (w_n) **2 marks**

v. Express w_n in terms of n. **2 marks**

b) Show that $\forall n \in \mathbb{N}$, we have $u_n = \dfrac{2n-1}{2^n}$ **2 marks**

c) Let $S_n = \sum\limits_{k=0}^{n} u_k$. Prove by mathematical induction that $\forall n \in \mathbb{N}$ $S_n = 2 - \dfrac{2n+2}{2^n}$ by mathematical induction.

3 marks